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Generalized I-Contractions and Pointwise R-Subweakly Commuting Maps

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Abstract The existence of common fixed points and invariant approximations for pointwise R-subweakly commuting and compatible maps is established. Our results unify and generalize various known results to a more general class of noncommuting mappings.

Keywords common fixed point, pointwise R-subweakly commuting maps, tangential maps, diminishing orbital diameters, invariant approximation

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1 Introduction and Preliminaries

Let M be a subset of a normed space $(X, \ \cdot \ )$. The set $P_M(u) = \{x \ \ M : \ x-u \ = \mathrm{dist}(u, M)\}$ is called the set of best approximants to u X out of M, where $\mathrm{dist}(u, M) = \inf\{ \ y-u \ : y \ \ M\}$. We denote by $_0$ the class of closed convex subsets of X containing 0. [1–2] For M $_0$, we define $M_u = \{x \ \ M : \ x \ \leq 2 \ u \ \}$. It is clear that $P_M(u) \ \ M_u \ \ _0$. The diameter of M is denoted and defined by $\delta(M) = \sup\{ \ x-y \ : x, y \ \ M\}$. A mapping $I : X \to X$ has diminishing orbital diameters (d.o.d.) [3] if, for each x X, $\delta(O(x)) < \infty$ and whenever $\delta(O(x)) > 0$, there exists $n = n_x$ N such that $\delta(O(x)) > \delta(O(I^n(x)))$, where $O(x) = \{I^k(x) : k \ \ N \ \ \{0\}\}$ is the orbit of I at x and $O(I^n(x)) = \{I^k(x) : k \ \ N \ \ \{0\}$ and $k \geq n\}$ is the orbit of I at $I^n(x)$ for n N $\{0\}$. Let $I : M \to M$ be a mapping. A mapping $T : M \to M$ is called an I-contraction if there exists $0 \leq k < 1$ such that $\ Tx - Ty \ \leq k \ Ix - Iy \ $, for each x, y M. The mapping T is said to be I-continuous [4] if $Ix_n \to Ix$ implies $Tx_n \to Tx$ whenever $\{x_n\}$ is a sequence in M and x M. The set of fixed points of T (resp. I) is denoted by F(T) (resp. F(I)). A point x M is a coincidence point (common fixed point) of I and T if $Ix = Tx$ $(x = Ix = Tx)$. The set of coincidence points of I and T is denoted by C(I, T). The set M is called q-starshaped with q M, if the segment $[q, x] = \{(1-k)q + kx : 0 \leq k \leq 1\}$ joining q to x is contained in M for all x M.

Two self-maps I and T of a metric space (X, d) are called: (1) commuting if $TIx = ITx$ for all x X; (2) compatible [3] if $\lim_n d(TIx_n, ITx_n) = 0$ whenever $\{x_n\}$ is a sequence such that $\lim_n Tx_n = \lim_n Ix_n = t$ for some t in X; (3) nontrivially compatible [3] if I and T are compatible and do have a coincidence point; (4) weakly compatible (or partially commuting [4]) if they commute at their coincidence points, i.e., if $ITx = TIx$ whenever $Ix = Tx$; (5) noncompatible [5] if there exists some sequence $\{x_n\}$ in M such that $\lim_n Tx_n = \lim_n Ix_n = t$

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