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Analytic approximation of solutions of the forced Duffing equation with integral boundary conditions

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Abstract

A sequence of approximate solutions converging monotonically and quadratically to the unique solution of the forced Duffing equation with integral boundary conditions is obtained. We also establish the convergence of order k $(k \geqslant 2)$ for the sequence of iterates. The results obtained in this paper offer an algorithm to study the various practical phenomena such as prediction of the possible onset of vascular diseases, onset of chaos in speech, etc. Some interesting observations are presented.

MSC: 34B10; 34B15

Keywords: Duffing equation; Integral boundary conditions; Quasilinearization; Quadratic convergence; Higher order convergence

1. Introduction

Integral boundary conditions for evolution problems have various applications in chemical engineering, thermoelasticity, underground water flow and population dynamics, see for example [19,25,47,48]. Vascular diseases such as atherosclerosis and aneurysms are becoming frequent disorders in the industrialized world due to sedentary way of life and rich food. Causing more deaths than cancer, cardiovascular diseases are the leading cause of death in the world. In recent years, computational fluid dynamics (CFD) techniques have been used increasingly by researchers seeking to understand vascular hemodynamics. Most of the CFD-based hemodynamic studies so far have been conducted to represent *in vitro* conditions within restrictive assumptions. These studies under *in vitro* conditions are well suited to investigate basic phenomena related to fluid dynamics in vessels models but are not fully representative of actual patient hemodynamic conditions. In fact, CFD methods possess the potential to augment the data obtained from *in vitro* methods by providing a complete characterization of hemodynamic conditions (blood velocity and pressure as a function of space and time) under precisely controlled conditions. However, specific difficulties in CFD studies of blood flows are related to the boundary conditions. It is now recognized that the blood flow in a given district may depend on the global dynamics of the whole circulation. Consequently, it is sometimes necessary to couple the 3D blood flow solver to a low order model for the entire vascular system [26]. A second difficulty is related to the limitations of the existing *in vitro* anemometry techniques. Indeed, the space resolution is far too coarse to tackle even the largest scales

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